Zeszyty Naukowe WWSI, No 15, Vol. 10, 2016, s. 7-17

On EDF scheduler with the exponential deadlines

Robert Janowski*[1], Andrzej Janowski[2]

[1] Warsaw School of Computer Science

[2] Independent scholar

Abstract

This work deals with the performance evaluation of EDF (Earliest Deadline First) packet scheduler with two classes. The primary metric of interest is the mean sojourn time for each class. The system is composed of two classes (two queues) with Poisson input to each of them, deterministic service times and exponentially distributed deadline values. The model is analysed as an embedded Markov chain at the instants of packet departures from the service. The solution i.e. the joint probability distribution of the number of packets in each queue is obtained using the matrix approach. The metrics such as the mean sojourn time or the mean number of packets in the system for each class are directly obtained from this joint probability distribution.

Keywords – Earliest Deadline First, scheduling, performance evaluation, Markov chains

1. Introduction

At the output port of a packet network node with more than one traffic class (appropriately marked packet streams and their associated queues) there is always an issue

* E-mail: rjanowski@poczta.wwsi.edu.pl

Manuscript received June 21, 2016.

what order the packets from different classes should go to the service. The algorithm determining the order the packets from different classes (queues) are taken to the service i.e. the transmission at the output port is called the scheduling algorithm. There exist several scheduling algorithms proposed for the packet service at the network nodes. Among them the most popular are FIFO (First In First Out), PQ (Priority Queueing) [1], WFQ (Weighted Fair Queueing) [2], EDF (Earliest Deadline First) [3].

Each of these algorithms was designed to meet particular requirements, e.g. FIFO algorithm which stores all packets from different streams (classes) in one queue was designed to achieve implementation simplicity. PQ algorithm was designed to assure a minimum queueing delay for the packet stream (class) served with the highest priority. WFQ was designed to provide fair bandwidth sharing (i.e. the ability to utilize the unused bandwidth) among different classes with the guarantees of the minimum bandwidth for each class. Finally EDF was designed to meet some delay requirements imposed for each class.

Depending on the scheduling algorithm and the values of its parameters the packets from different classes experience different delays and build up queues of different sizes. The measures of packet delays and queue sizes are the basic metrics of interest [4], [5] because they are used for resource dimensioning and performance evaluation at the network nodes. In this work we focus on the mean packet delays for each class served with EDF algorithm.

There have been already published some papers about the performance of EDF scheduler. They differ in assumptions, targets and methods of providing the solution. In [6] the authors derive bounds on the packet delay for packet flows with traffic characteristics bounded by some deterministic values. The paper [7] studies the analytical method to approximate the fraction of jobs missing their deadlines (the assumed performance measure) when earliest-deadline-first (EDF) scheduling policy is used. The deadlines have general distribution the input is modelled as Poisson stream and the service times have the exponential distribution.

The authors of [8] derive the stochastic bounds for the probability distribution function of the packet delay in case of a number of multiplexed packet flows each modelled as Markov Modulated On-Off source. They use martingale and sample-path approach.

In [9] EDF scheduler with the shaped input traffic i.e. traffic flows modelled as Exponentially Bounded Burstiness is analysed. The authors derive the stochastic bounds on the probability distribution of the end-to-end delay for traffic shaper elements and EDF scheduler with deterministic deadlines.

In [10] the large deviation principle and the effective bandwidth theory are used to develop an analytical model which is able to predict the deadline violation probability of individual classes in Earliest Deadline First (EDF) scheduler with deterministic deadline values.

The rest of the paper is organized as follows: In section 2 we provide the analysis of the modelled system. We start with the description of the modelled system and EDF algorithm in subsection 2.1. In subsection 2.2 we explain the analytical method for the system evaluation. In subsection 2.3 we provide the formulas for the mean sojourn and waiting times. In section 3 we provide numerical results with the comparison to the results of PQ algorithm. Section 4 summarizes the work.

2. Analysis of the system

2.1 Description of the modelled system

We analyse the output port in a packet network node with EDF scheduler and two traffic classes (two packet streams). Packets of each class are stored in a separate buffer i.e. they build separate queues. Packets belonging to different classes are scheduled to the service according to EDF algorithm with exponential deadlines. It means that on a packet arrival the packet is assigned a deadline random value drawn from the exponential distribution.

For each packet waiting in its queue its deadline is decreased until it goes into the service. When the currently served packet departs from the system (its transmission ends) and the next packet is to be chosen for the service (i.e. transmission) the current deadline values of packets in the head of line position of each queue are compared and the packet with the smallest deadline value is taken to the service. In case one of these queues is empty the packet from the non-empty queue is scheduled for the transmission no matter what its deadline value is.

The above described system can be modelled as two queues (one for packets of traffic class #1 and the other for packets of traffic class #2) with a single service station (modelling the output port) as shown in Figure 1.

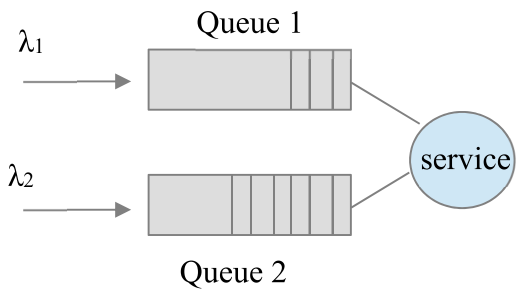

Figure 1. The model of the system with two classes (two queues)

The packets of each class arrive to the system according to Poisson process [11] with intensities λ_1 and λ_2 for class #1 and class #2 respectively. The packets of each class have a constant length such that their transmission times are t_1 and t_2 for class #1 and class #2 respectively. The values of deadlines are drawn from the exponential distribution with parameters μ_1 and μ_2 for class #1 and class #2 respectively.

We observe the system state at the time instants (the epochs) just after the departure of a packet from the service. The system state just after the n-th packet departure we define as a vector of 3 variables: M_n – the number of packets type #1 in the system, N_n – the number of packets type #2 in the system, O_n – the type of packet just served. The random variable O_n takes the value 1 for a packet type #1 and the value 2 for a packet type #2. The time instants of the observation of the system state, the packet arrivals as well as the packet services are depicted in the Figure 2.

The vector S_n forms a Markov chain [12] i.e. the state of the system at the (n+1)-th epoch can be fully determined based on the information about the system state at the n-th epoch.

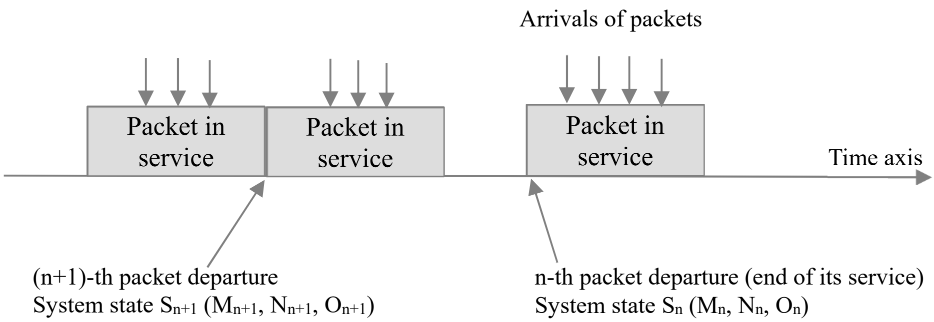

Figure 2. The time evolution of the system state

2.2.1. Analysis of the probability distribution of the number of packets in the system

In order to characterize the time evolution of the system state it is enough to determine the transition probabilities from state S_n to the state S_{n+1}. i.e.

$$\Pr ob\{S_{n+1}(m,n,o) \mid S_n(i,j,k)\} \tag{1}$$

First we notice that the deadline values are random variables X_1 and X_2 for class #1 and class #2 respectively with the exponential distribution i.e. their probability density functions are given by the following formulas:

$$f_1(x) = \mu_1 e^{-\mu_1 x} \tag{2}$$

$$f_2(x) = \mu_2 e^{-\mu_2 x} \tag{3}$$

The exponential distribution function posses the memoryless property [11] which means that the probability distribution of the remaining time doesn't depend on the time you have already spent in the system. The proof is outlined below:

$$\Pr ob\{X > t+x \mid X > t\} = \frac{\Pr ob\{X > t+x \mid X > t\}}{\Pr ob\{X > t\}} = \frac{\Pr ob\{X > t+x\}}{\Pr ob\{X > t\}} = \frac{e^{-\lambda(t+x)}}{e^{-\lambda t}} = e^{-\lambda x} = \Pr ob\{X > x\} \tag{4}$$

In our case it means that at the moment of scheduling the next packet for the service it is enough to compare the deadlines values as if they were drawn at that instant. In other words in order to decide which packet (class #1 or class #2) should be scheduled for the transmission it is enough to calculate the probabilities:

$$\Pr ob\{X_2 \le X_1\} \tag{5}$$

and

$$\Pr ob\{X_1 \le X_2\} \tag{6}$$

Each time there is at least one packet present in each queue in the system the packet class #1 is scheduled with the probability (6) and the packet class #2 is scheduled with probability (5).

The probability given in (5) can be determined as follows:

$$\Pr ob\{X_2 - X_1 \le u\} = F_U(u) = \int_0^{\infty} (\mu_1 e^{-\mu_1 x} \int_0^{x+u} \mu_2 e^{-\mu_2 y} dy)dx = ... = 1 - \frac{\mu_1}{\mu_1 + \mu_2} e^{-\mu_2 u} \qquad (7)$$

Thus,

$$\Pr ob\{X_2 \le X_1\} = F_U(0) = \frac{\mu_2}{\mu_1 + \mu_2} \qquad (8)$$

In a similar way the probability given in (6) can be determined to be:

$$\Pr ob\{X_1 \le X_2\} = \frac{\mu_1}{\mu_1 + \mu_2} \qquad (9)$$

Regarding the number of the packets in the system at the (n+1)-th epoch i.e. M_{n+1} and N_{n+1} for class #1 and class #2 respectively it is simply the number of packets observed in the system at the previous n-th epoch plus the number of packets that have arrived during the service time (to each queue separately) minus the one packet that has departed from the service. If the departing packet belonged to class #1 the queue #1 is decreased. If the departing packet belonged to class #2 the queue #2 is decreased. If the departing packet left the system empty (both classes are empty) then no queue is decreased. This behaviour can be summarize in the following set of equations:

$$S_{n+1}(M_{n+1}, N_{n+1}, O_{n+1}) | = \left\{ \begin{array}{ll} M_{n+1} = M_n - 1 + A_{n+1,1} & if \quad M_n > 0 \wedge O_n = 1 \\ M_{n+1} = M_n + A_{n+1,1} & if \quad (M_n > 0 \wedge O_n = 2) \vee M_n = 0 \\ N_{n+1} = N_n - 1 + A_{n+1,2} & if \quad N_n > 0 \wedge O_n = 2 \\ N_{n+1} = N_n + A_{n+1,2} & if \quad (N_n > 0 \wedge O_n = 1) \vee N_n = 0 \end{array} \right\} \qquad (10)$$

where $A_{n+1,i}$ (i=1,2) denotes the random variable describing the number of arrived packets type 1 or 2 to the respective queue during the service time of a packet that has departed from the system at the (n+1)th epoch.

Taking into account the set of equations (10) and the probabilities of the next packet to be scheduled for the service given in equations (8) and (9) the transition probabilities from state $S_n(i,j,k)$ to the state $S_{n+1}(m,n,o)$ can be calculated according to the formulas given in (11):

$$\Pr ob\{S_{n+1}(m,n,o) \mid S_n(i,j,k)\} =$$
$$= \begin{cases} \Pr ob\{A_{n+1,1} = m-i+1\} \Pr ob\{A_{n+1,2} = n-j\} \dfrac{\mu_1}{\mu_1+\mu_2} & if \quad i \neq 0 \wedge j \neq 0 \wedge k = 1 \wedge o = 1 \\ \Pr ob\{A_{n+1,1} = m-i+1\} \Pr ob\{A_{n+1,2} = n-j\} \dfrac{\mu_2}{\mu_1+\mu_2} & if \quad i \neq 0 \wedge j \neq 0 \wedge k = 1 \wedge o = 2 \\ \Pr ob\{A_{n+1,1} = m-i\} \Pr ob\{A_{n+1,2} = n-j+1\} \dfrac{\mu_1}{\mu_1+\mu_2} & if \quad i \neq 0 \wedge j \neq 0 \wedge k = 2 \wedge o = 1 \\ \Pr ob\{A_{n+1,1} = m-i\} \Pr ob\{A_{n+1,2} = n-j+1\} \dfrac{\mu_2}{\mu_1+\mu_2} & if \quad i \neq 0 \wedge j \neq 0 \wedge k = 2 \wedge o = 2 \\ \Pr ob\{A_{n+1,1} = m-i\} \Pr ob\{A_{n+1,2} = n-j+1\} * 0 & if \quad i = 0 \wedge j \neq 0 \wedge (k = 1 \vee k = 2) \wedge o = 1 \\ \Pr ob\{A_{n+1,1} = m-i\} \Pr ob\{A_{n+1,2} = n-j+1\} * 1 & if \quad i = 0 \wedge j \neq 0 \wedge (k = 1 \vee k = 2) \wedge o = 2 \\ \Pr ob\{A_{n+1,1} = m-i+1\} \Pr ob\{A_{n+1,2} = n-j\} * 1 & if \quad i \neq 0 \wedge j = 0 \wedge (k = 1 \vee k = 2) \wedge o = 1 \\ \Pr ob\{A_{n+1,1} = m-i+1\} \Pr ob\{A_{n+1,2} = n-j\} * 0 & if \quad i \neq 0 \wedge j = 0 \wedge (k = 1 \vee k = 2) \wedge o = 2 \\ \Pr ob\{A_{n+1,1} = m-i\} \Pr ob\{A_{n+1,2} = n-j\} \dfrac{\lambda_1}{\lambda_1+\lambda_2} & if \quad i = 0 \wedge j = 0 \wedge (k = 1 \vee k = 2) \wedge o = 1 \\ \Pr ob\{A_{n+1,1} = m-i\} \Pr ob\{A_{n+1,2} = n-j\} \dfrac{\lambda_2}{\lambda_1+\lambda_2} & if \quad i = 0 \wedge j = 0 \wedge (k = 1 \vee k = 2) \wedge o = 2 \end{cases} \tag{11}$$

In (11) the transition probabilities have been expressed only in terms of the probability of new packet arrivals during the service time of a packet. Since in subsection 2.1 we assumed the arrivals form Poisson process with intensities λ_1 and λ_2 for class #1 and class #2 respectively the probability that are '*x*' class #1 packet arrivals is given by (12) [11]:

$$\Pr ob\{A_{n+1,1} = x\} = \frac{(\lambda_1 t)^x}{x!} e^{-\lambda_1 t} \tag{12}$$

and the probability that there are '*x*' class #2 packet arrivals is given by (13):

$$\Pr ob\{A_{n+1,2} = x\} = \frac{(\lambda_2 t)^x}{x!} e^{-\lambda_2 t} \tag{13}$$

In equations (12) and (13) the parameter *t* should be substituted with t_1 or t_2 depending on the type of packet departing from the system at (n+1)th epoch.

Equations (11), (12) and (13) let us calculate the transition probabilities from state $S_n(i,j,k)$ to the state $S_{n+1}(m,n,o)$. These probabilities form so called transition matrix denoted T.

Solving the matrix equation given in (14) provides us with the solution i.e. Π vector which is the joint steady-state probability distribution of the three random variables (M,N,O):

$$\Pi = \Pi * T \qquad (14)$$

The probability distribution Π_1 of the number of packets class #1 (or Π_2 of the number of packets class #2) in the system can be calculated as the marginal distribution of the solution Π. Using the probability distributions Π_1 or Π_2 one can easily calculate the mean values.

2.3 Analysis of the mean waiting and mean sojourn times

As pointed out in section 1 it is especially interesting how EDF scheduler and the values of its parameters (μ_1 and μ_2) impact the packet delays for each class.
The mean value of the sojourn time E(T) (the time from the packet arrival till its departure from the system) can be calculated based on Little's formula [12] which states that the mean number of packets in the system E(N) equals the mean sojourn time E(T) multiplied by the packet arrivals intensity λ:

$$E(N) = \lambda E(T) \qquad (15)$$

The mean waiting time E(W) can be obtained from the mean sojourn time E(T) since their difference is the mean service time E(S):

$$E(W) = E(T) - E(S) \qquad (16)$$

3. Numerical results

The results have been obtained using SageMath [13] Open Source tool. The mean sojourn times for two packet classes ($E(T_1)$ and $E(T_2)$) with packet arrival intensities equal to $\lambda_1=\lambda_2=0.25$ and packet service times $t_1=t_2$ equal to 1 are presented on Figure 3. The mean sojourn times are expressed in units equal to the service (transmission) time of a single packet ($t_1=t_2=1$). On the same Figure 3 there are also results for PQ scheduler with two classes and the same traffic parameters (arrival intensities and packet service times) as for EDF scheduler. For PQ scheduler it is assumed that packets of class #2 have higher priority than packets of class #1.

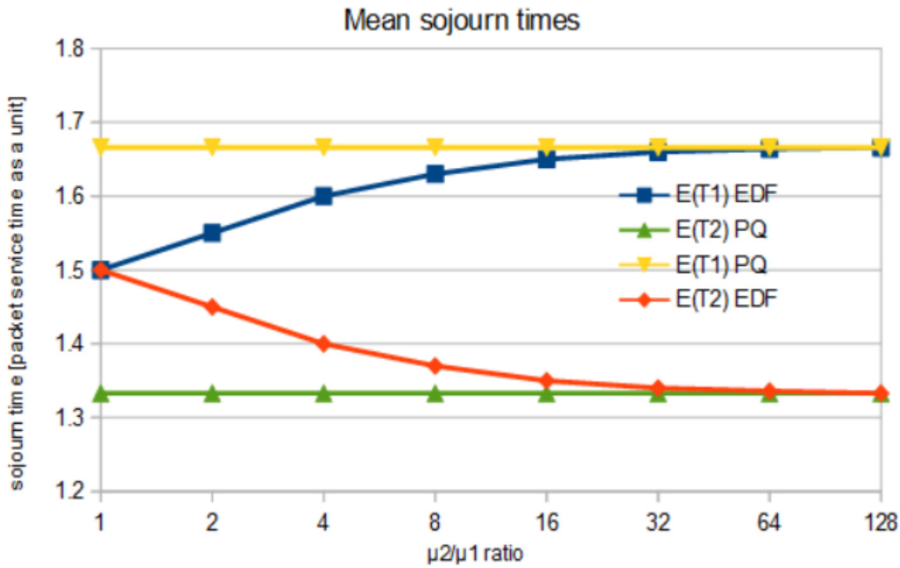

Figure 3. Mean sojourn times for two packet classes with EDF or PQ scheduler as a function of μ_2/μ_1 ratio

The results obtained for EDF scheduler have been referenced to the results of PQ scheduler. In this comparison PQ scheduler might be considered as a limit case i.e. it is equivalent as setting the value of μ_1 or μ_2 parameter to infinity. In PQ scheduler with two traffic classes the packets of the lower priority class will be served only when there are no higher priority packets in the system. Similar behaviour would be observed with μ_1 (or μ_2) parameter set to infinity. Then every time there is a decision what packet should be served (class #1 or class#2) the packets belonging to the class with μ parameter set to infinity would be chosen. This is due to the fact that with μ_1 (or μ_2) set to infinity the probability given in equation (9) (equation (8) respectively) tends to 1. From the obtained results we can conclude when the ratio of μ_2/μ_1 approaches 64 EDF scheduler behaves almost as PQ scheduler with class #2 having the higher priority.

On the other hand when both classes have the same parameters including μ_2 and μ_1 responsible for deadline values then packets from both classes experience the same delays. This delay is the same as for FIFO scheduler since there is no differentiation between the classes as they have all the same parameter values. On the Figure

3 this is the point for μ_2/μ_1 equal 1. When the μ_2/μ_1 ratio increases the mean waiting and sojourn times of these classes diverge from themselves.

4. Summary

In this work we have modelled and solved the system with two traffic classes (queues) and EDF scheduler with deadline values following the exponential distribution. We have defined the system state as a set of three random variables and described the evolution of the system state as a Markov chain. We have provided the numerical results for a given set of parameter values and showed the behaviour of EDF scheduler can approach the behaviour of PQ scheduler when the ratio of μ values (either μ_2/μ_1 or μ_1/μ_2) tends to infinity. On the other hand, if the parameter values are the same for both classes EDF scheduler behaves like FIFO. We can conclude that adjusting the value of μ_2/μ_1 ratio we can control the differentiation of the delay experienced by packets of each class and bias it toward FIFO or the opposite direction i.e. toward PQ scheduler.

Literature

[1] Jaiswal N.K., *Priority queues*, Academic Press, New York, 1968

[2] Parekh A.K., Gallager R.G., *A generalized processor sharing approach to flow control in integrated services networks*, "IEEE/ACM Transaction on Networking" Vol. 1, No. 3, June 1993

[3] Stankovic J.A., Spuri M., Ramamritham K., Buttazzo G.C., *EDF scheduling for Real Time systems*, Springer International Series in Engineering and Computer Science, 1998

[4] ITU-T Recommendation Y.1541, *Network performance objectives for IP-based services*, ITU, May 2002

[5] ITU-T Recommendation Y.1540, *Internet Protocol data communication service – IP packet transfer and availability performance parameters*, ITU, January 2002

[6] Bui D., *Revision of a Non-Preemptive EDF Packet Scheduling Algorithm*, Technical Report No. UCB / EECS-2009-50, http://www.eecs.berkeley.edu/Pubs/TechRpts/2009/EECS-2009-50.html, April 24, 2009

[7] Kargahi M., Movaghar A., *A Method for Performance Analysis of Earliest-Deadline-First Scheduling Policy*, "The Journal of Supercomputing" 37, 2006

[8] Ciucu F., Poloczek F., Schmitt J., *Sharp Bounds in Stochastic Network Calculus*, “SIGMETRICS '13” June 2013
[9] Biton E., Orda A., *QoS Provision with EDF Scheduling, Stochastic Burstiness and Stochastic Guarantees*, proceedings of 18th International Teletraffic Congress, ITC18, September 2003
[10] Scriba S.M., Takawira F., *Packet violation probability analysis of the Earliest Deadline First scheduler*, Proceedings of Southern African, 2004
[11] Gross D, Harris C., *Fundamentals of queueing theory*, 3rd edition, Wiley, Hoboken, 2002
[12] Kleinrock L., *Queueing Systems*, Vol. 1 & 2, Wiley, Hoboken, 1975
[13] SageMath, https://www.sagemath.org/